

The lost problems of the Chang Ch'iu-chien Suan Ching, a fifth-century Chinese mathematical manual

by Ho Peng Yoke^[1]

(University of Malaya, Kuala Lumpur, Malaysia)

Among the few traditional Chinese mathematical texts that still survive today is the *Chang Ch'iu-chien Suan Ching*^[2] (*Mathematical Manual of Chang Ch'iu-chien*), written by CHANG CH'IU-CHIEN^[3] some time between the years A.D. 468 and A.D. 486¹. At the beginning of the Ch'ing period Wang Chieh^[4] had in his possession a Southern Sung (1127 to 1279) printed copy of this text. Later it was acquired by Mao Chin^[5], and now it is preserved in the Shanghai Library. During the first year of the K'ang-hsi reign-period (1662) Mao I^[6] made a copy from the Sung edition, and this later entered the T'ien Lu Lin Lang Ko^[7] Palace-Library². On this were based the edition of the *Chang Ch'iu-chien Suan Ching* in the *Ssü K'u Ch'üan Shu* Collection and that in K'UNG CHI-HAN'S^[8] (1739/1783) *Wei Po Hsieh Ts'ung Shu*^[9] Collection (1773). The texts of the *Chang Ch'iu-chien Suan Ching* later included in the *Chih Pu Tsu Chai Ts'ung Shu*^[10] Collection (1777)³, the *Ku Chin Suan Hsüeh Ts'ung Shu*^[11] Collection (1898) and the *Wan Yu Wên K'u* (1937) all followed the *Wei Po Hsieh Ts'ung Shu*. The first series of the *Ts'ung Shu Chi Ch'êng* (1939) reproduces the text from the *Chih Pu Tsu Chai Ts'ung Shu*. Textual comparisons have been recently made by Ch'ien Pao-tsung^{[12]4}.

The text of the *Chang Ch'iu-chien Suan Ching* that we now have in all the versions mentioned above is not complete. The missing portion comes at the end of the Second Chapter and the beginning of the Third Chapter in all these cases. The end of the Second Chapter in the *Chih Pu Tsu Chai Ts'ung Shu* edition, for example consists of one or more blank leaves, beginning from page 22a, while the beginning of the Third Chapter consists of two blank leaves, representing pages 1a, 1b, 2a and 2b. Page 1a of the Third Chapter contains five printed lines giving the title of the book, the name of the author, the name of the commentator CHÊN LUAN^[13] (fl. A.D. 560/A.D. 580) and the name of the writer of the workings, LIU HSIAO-SUN^[14] (A.D. 576/A.D. 625). It is the purpose of this article to discuss the missing part of the text.

[1] 何丙郁
[2] 張邱建算經
[3] 張邱建
[4] 王杰
[5] 毛晉

[6] 毛扆
[7] 天祿琳琅閣
[8] 孔繼涵
[9] 徵及樹叢書
[10] 知不足齋叢書

[11] 古今算學叢書
[12] 錢寶琮
[13] 甄鸞
[14] 劉孝孫

The fifty-fourth problem in the *Chang Ch'iu-chien Suan Ching* deals with the finding of the sagitta of a segmental area (*hu t'ien*)^[15], given the area and the length of the chord. The whole Working of this example is missing, and it must have been contained in page 22a and page 22b of Chapter Two. Similarly, the last few words of the given 'Method' must have been formerly contained in page 22a. This is rather unfortunate, as the 'Working' is supposed to explain the *tai tsung k'ai lang*^[16] method employed by Chinese mathematicians for solving quadratic equations and equations of higher degree⁵. What still remains of this Problem says:

"Given a segment with a chord $68 \frac{3}{5} pu$ ^[17] long and area $2 mou$ ^[18] $34 \frac{32}{45} pu$, find its sagitta.

The 'Answer' says: The sagitta is $12 \frac{2}{3} pu$.

The 'Method' says: Place twice the segmental area expressed in (square) *pu* (on the calculation board) in the *shih*^[19] (position).

Take the number of *pu* in the chord as the *tsung*^[20] . . . "

The 'Method' is incomplete and the 'Working' is missing altogether. Nonetheless, one can see that Chang Ch'iu-chien must have in mind the same formula used by Liu Hui^[21] some two centuries earlier for a relationship between the area *A*, the chord *c* and the sagitta *s* of a segment, i.e.

$$A = \frac{1}{2} s (s + c)$$

This formula is of course not quite correct⁶. However, it is the method of solving the quadratic equation involved that is of particular interest to us. It must be also on the basis of this same formula that Li Yen amended the last fraction of the segmental area from $\frac{32}{45}$ to $\frac{31}{45}$, assuming an exact solution⁷.

The 'Working' has been reconstructed by Ku KUAN-KUANG^[22] (1799/1862) in his *Chiu Shu Ts'un Ku*^[23]⁸. The passage is as shown in Figure 1. It reads as follows:

"The Method says: Place twice the segmental area expressed in (square) *pu* (on the counting board) in the *shih* (position). Take the number of *pu* of the chord as the *tsung*. The sagitta is obtained by means of (the method of) *k'ai p'ing lang*^[24] (lit. 'extraction of square root'). (Note:) the last sentence above has been reconstructed from the context."

"The 'Working' says: Use the equivalent of *mou* and *pu* measures and convert 2 *mou* to 480 *pu*. Add to this the other *pu* values (i.e. $34 \frac{32}{45}$). Then change into improper fraction by multiplying (514) by the denominator (45) and add to it the numerator (32), (thus) obtaining ($\frac{23162}{45}$ with a numerator) 23162. Doubling this value gives 46324, which is (to be placed in) the *shih* (position on the counting-board). Also multiply the chord of 68 *pu* by the denominator 45, giving 3600 (and place this number somewhere) in a top position (on the counting-board), and multiply the numerator 3 by 9, giving 27. (Note that) 45 is equal to 9 times 5, hence (the numerator 3 in the fraction $\frac{3}{5}$ is) multiplied by 9 (instead of multiplying the fraction itself by 45).

[15] 弧田

[16] 帶從開方

[17] 步

[18] 畝

[19] 實

[20] 從

[21] 劉徽

[22] 顧觀光

[23] 九數存古

[24] 開平方

今有弧田弦六十八步五分步之三爲田二畝
三十四步四十五分步之三十二問矢幾何

答曰矢一十二步三分步之二

術曰置田積步倍之爲實以弦步數爲從開平

方除之得矢以上七補字

草曰以畝法通二畝爲四百八十步加入

餘步又通分內子得二萬三千一百六十

二倍之得四萬六千三百二十四爲實又

以分母四十五乘弦六十八步得三千六

十于上又九因分子三千四百五爲五之得

二十七併入上位得三千八百七爲九因得

四十五爲正隅開平方得十二步餘實二

千八百方四千一百六十七隅四十五以

隅四十五爲母乘餘實得十二萬六千爲

實方不動隅定爲一開平方得三十爲子

不盡九百子母各以十五約之爲三分步

之二十合問此草原本脫去今以

意補故低二格別之

Figure 1 Reconstruction by Ku Kuan-kuang

(The value 27) is added to (the figure 3600 in) the top position, giving the number 3087 which is (then placed in) the *tsung fang*^[25] (position on the counting-board). (The number) 45 is (placed in) the *chêng yü*^[26] (position on the counting-board). Applying (the method of) *k'ai p'ing fang* (lit. 'extraction of square root') (the result) 12 *pu* is obtained, with remainders 2800 in the *shih* (position), 4167 in the (*tsung fang*) (position) and 45 in the (*chêng*) *yü* (position). Taking (the value) 45 in the (*chêng*) *yü* (position) as the denominator, multiply the remainder in the *shih* (position) by this number to give 126,000 which is (now) taken as the *shih*. The (number in the *tsung fang*) (position) is left unchanged, while the (number in the *chêng*) *yü* (position) is fixed as unity. Applying (the method of) *k'ai p'ing fang* (lit. *extraction of square root*) one obtains a numerator 30 (i.e. a fraction $\frac{30}{45}$). (Note that) the remainder 990 is left (on the counting-board). Dividing the numerator and the denominator by 15 (the fraction) becomes $\frac{2}{3}$ *pu*. Hence the answer (12 $\frac{2}{3}$ *pu*). (Note that) this Working is missing in the original text, but is here being reconstructed from the context. Hence it is printed two rows lower than the text to distinguish it."

It can be seen that KU KUAN-KUANG first expressed the problem in the form of a quadratic equation:

$$45s^2 + 3087s - 46324 = 0$$

He gives the result without explaining the process. From his result we can deduce that he has used a process well-known to the thirteenth-century Chinese mathematicians Ch'in Chiu-shao^[27], Li Chih^[28] and Yang Hui^[29] and later developed independently in the early nineteenth century by Ruffini and Horner. The process would be as follows:

<i>chêng yü</i>	<i>tsung fang</i>	<i>shih</i>	
45	3087	—46324	10
	<u>450</u>	<u>35370</u>	
	3537	<u>—10954</u>	
	<u>450</u>		
	<u>3987</u>		

<i>chêng yü</i>	<i>tsung fang</i>	<i>shih</i>	
45	3987	—10954	2
	<u>90</u>	<u>8154</u>	
	4077	<u>—2800</u>	
	<u>90</u>		
	<u>4167</u>		

A misprint appears in KU KUAN-KUANG's notes on finding the fraction. After converting the equation

$$45s^2 + 4167s - 2800 = 0$$

into the form

$$y^2 + 4167y - 126,000 = 0$$

[25] 從方

[26] 正隅

[27] 秦九韶

[28] 李治

[29] 楊輝

where $s = \frac{y}{45}$.

The process of finding y continues as follows:—

<i>chêng yū</i>	<i>tsung tang</i>	<i>shih</i>	
1	4167	126000	30
	30	125910	
	-----	-----	
	4197	90	
	30	-----	
	-----	-----	
	4227		

the value $y = 30$ or $s = \frac{30}{45} = \frac{2}{3}$ is obtained.

The remainder should read '90' and not '990' as stated in KU KUANG-KUANG'S notes.

The process used above was already in use during the thirteenth century in China and was described both by Ch'in Chiu-shao and Yang Hui. The former, for example, called it the *lien chih t'ung t'i shu*^[30]. It is of course possible to obtain the same result by applying the process of *lien chih t'ung t'i shu* from the very beginning by converting the equation

$$45s^2 + 3087s - 46324 = 0$$

into the form

$$y^2 + 3087y - 2084580 = 0$$

where $s = \frac{y}{45}$.

Li Yen is not entirely satisfied with KU KUANG-KUANG'S reconstruction of the 'Workings' in the problem on the segmental area in the *Chang Ch'iu-chien Suan Ching*⁹. Earlier he mentions the sad omission of the *tai tsung k'ai tang* method at this point in the text, and this method as we have seen, is only referred to by KU KUANG-KUANG as the *k'ai p'ing tang* method, without any explanation of the process¹⁰. Wang Ling and Needham say that it is possible to show that if the text of the *Chiu Chang Suan Shu*^[31] is very carefully followed, the essentials of the methods used by the Chinese for solving numerical equations of the second and higher orders and similar to that developed by Horner in 1819, are already there at a time which may be dated as of the first century B.C.¹¹. It may therefore be assumed that the method was known to Chang Ch'iu-chien in the A.D. fifth century and to Liu Hsiao-sun in the sixth and seventh centuries. In fact, this can be seen from Problem No. 51 in the *Chang Ch'iu-chien Suan Ching*, which gives the solution of a second degree equation in the form $x^2 - 127449 = 0$. It is worthwhile to give a detailed analysis of the process given.

The problem involves the finding of the side of a square farm with a given area 127449 (square) *pu*. The steps in the 'Working' are as follows:

Step 1

置前積步數於上

Place the given (number expressed in) number of (square) *pu* in an upper (row of the counting-board). (This is called the *shih*).

[30] 連枝同體術

[31] 九章算術

Step 2

借一算子於下

Make use of one counting-rod in the bottom (row of the counting-board in the furthest right-hand digit column).

Step 3

常超位一步至百止

This one counting-rod is always moved forward (from right to left) by steps of two places until it reaches the hundredth place-value (in this case). (This row is called the *hsia fa*)^[32].

Step 4

以上商置三百於積步之上

For (the top row above the *shih*, called) *shang shang*^[33], (first) put 300 above the given number (i.e. the *shih*).

Step 5

又置三萬於積步之下 下法之上 名曰方法 以命上商 三三如九

Then place 30,000 (obtained by multiplying the *hsia fa* by the first approximate root) below the given number (i.e. the *shih*) but above the *hsia fa*. This (row) is called the *fang fa*^[34]. Multiplying by (the digit of the first root in) the *shang shang* gives $3 \times 3(0,000) = 9(0,000)$.

Step 6

除九萬

90,000 is now divided from (the *shih*, and the same row is now replaced by the remainder)¹².

Step 7

又倍方法一退

Then double the *fang-fa* and move it back by one digit (to the right).

Step 8

↓法再退

The (counting-rod in the) *hsia fa* (row) is also moved backward (by one step).

Step 9

又置五十於上商之下

Again place 50 (i.e. the second figure of the root) below the *shang shang* row.

Step 10

又置五百於下之法上 名曰隅法

Then again place 500 (obtained by multiplying the *hsia fa* by the second figure of the root) above the *hsia fa*. This is called the *yü fa*^[35].

[32] 下法

[33] 上商

[34] 方法

[35] 隅法

Step 11

以方隅二法除實 餘有四千九百四十九

The *shih* is divided from (the product of the second figure of the root, i.e. 5 and the sum of) the *fang (fa)* and the *yü (fa)*, yielding a remainder 4949.

Step 12

又倍隅法以併方 得七千 退一等

Then double the *yü fa* and add this to the *fang (fa)*, giving 7,000. This is moved backward by one place.

Step 13

下法再退

The (counting-rod in the) *hsia fa* row is again moved backward (by one step).

Step 14

又置七於上商五十之下

Again place 7 (i.e. the third figure of the root) below (the second figure of the root) 50 in the *shang shang* row.

Step 15

又置七於下法之上 名曰隅法

Then again place 7 (obtained by multiplying the *hsia fa* by the third figure of the root) above the *hsia fa*. This is called the *yü fa*.

Step 16

以方隅法二除實 得合前問

Dividing the *shih* by the (product of the third root figure and the) sum of the *fang (fa)* and *yü (fa)*, the answer corresponding to the above question is obtained.

As a comparison the corresponding steps in the traditional Chinese method as described by Liu Hsiao-sun and the Ruffini-Horner method are shown in Figure 2 (a) to (e).

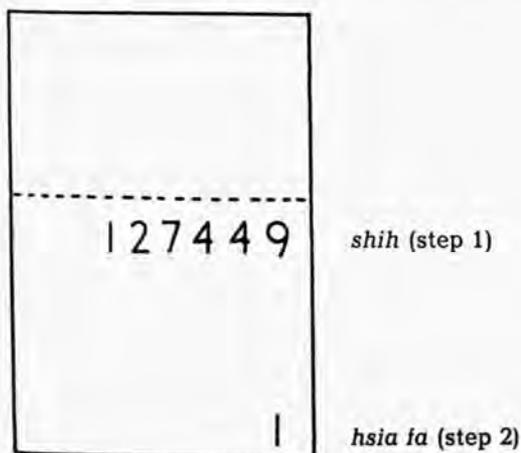
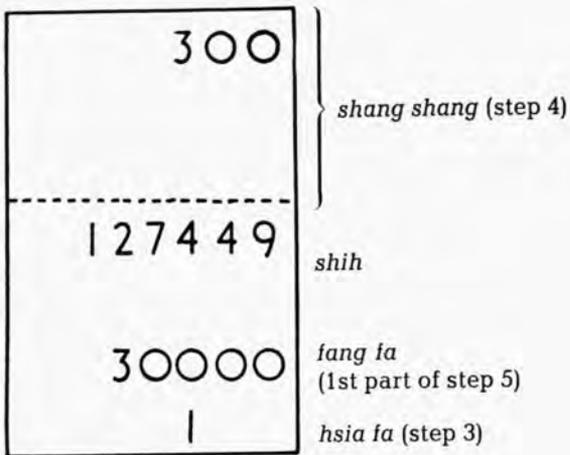
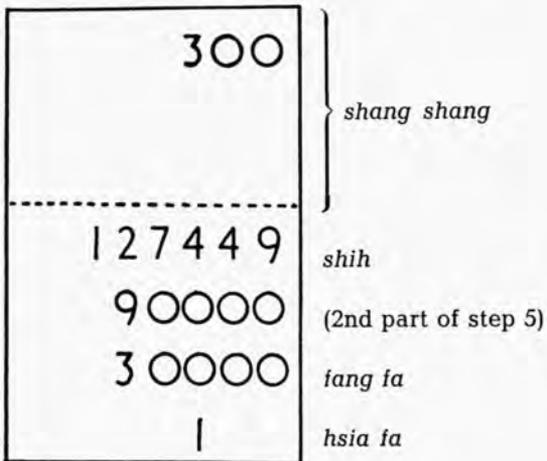


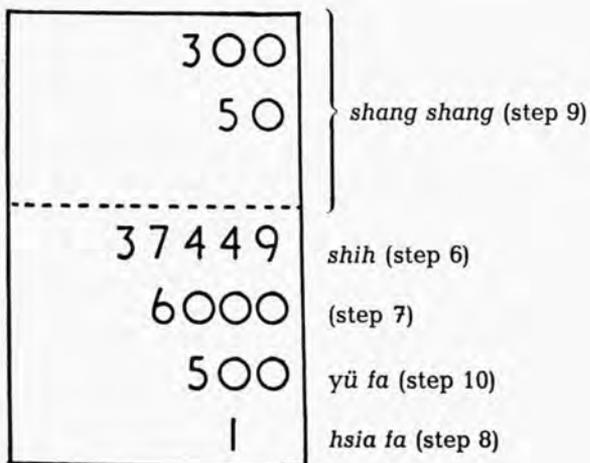
Figure 2
(a)



(b)



(c)



(d)

300	}	<i>Shang shang</i> (step 14)	(e)
50			
7			
4949		<i>shih</i> (step 11)	
700		<i>fang fa</i> (step 12)	
7		<i>yü fa</i> (step 15)	
1		<i>hsia fa</i> (step 13)	

(steps 2 & 3)

$$\begin{array}{r}
 1 \quad 0 \\
 \underline{300} \text{ (step 5)} \\
 300 \\
 \underline{300} \\
 600 \text{ (step 7)}
 \end{array}$$

(step 1)

$$\begin{array}{r}
 127449 \\
 \underline{90000} \text{ (step 5)} \\
 37449 \text{ (step 6)}
 \end{array}$$

300 (step 4)

(step 8)

$$\begin{array}{r}
 1 \quad 600 \\
 \underline{50} \text{ (step 10)} \\
 650 \\
 \underline{50} \\
 700 \text{ (step 12)}
 \end{array}$$

$$\begin{array}{r}
 37449 \\
 \underline{32500} \\
 4949 \text{ (step 11)}
 \end{array}$$

50 (step 9)

(step 13)

$$\begin{array}{r}
 1 \quad 700 \\
 \underline{7} \text{ (step 15)} \\
 707
 \end{array}$$

$$\begin{array}{r}
 4949 \\
 \underline{4949} \text{ (step 16)} \\
 0
 \end{array}$$

7 (step 14)

A striking resemblance between the two methods can be noticed.

An attempt is here made to improve on the re-construction of the lost text of KU KUAN-KUANG by incorporating the *tai-tsung k'ai fang* method and by copying the literary style of Liu Hsiao-sun, the author of the 'Working' in the *Chang Ch'iu-chien Suan Ching*. The reconstructed Chinese text as shown in Figure 3 reads:—

"Given a segment with a chord $68\frac{3}{5}$ pu long and area 2 mou $34\frac{32}{45}$ pu find its sagitta.

The 'Answer' says: The sagitta is $12\frac{2}{3}$ pu.

The 'Method' says: Place twice the segmental area expressed in (square) pu (on the calculation board) in the *shih* (position). Take the number of pu of

the chord as the *tsung*. The sagitta is obtained by means of (the method) *tsung êrh k'ai fang*^[36] (i.e. *tai tsung k'ai fang*).

The 'Working' says: Place (the number) 2 *mou* (on the calculation board) and multiply it by (the conversion factor) 240 (to bring it to) *pu*. Adding the number 34 gives 514 *pu*. Multiplying by the denominator 45 and adding the numerator 32 gives 23,162. Twice this number gives 46,324. Then place the value of the chord expressed in number of *pu*, (i.e.) 68 *pu* (on the counting-board), multiply it by the denominator 5 and add the numerator 3, giving 343. Dividing the denominator for the segmental area (expressed in square) *pu*, (i.e.) 45 by the denominator for the chord length (expressed) in *pu*, (i.e.) 5 gives 9. Multiplying 343 by 9 gives 3087, which is to be taken as the *tsung*. Then employ the method of *tsung fa k'ai fang* to extract the root."

"Place the above number 46,324 for the area in an upper (row of the counting-board). (This is called the *shih*). Place the denominator 45 separately in the bottom (row of the counting-board in the furthest right-hand digit column). This number is always moved forward (from right to left) by steps of two places until it reaches the hundredth place (in this case). (This row is called the *hsia fa*). Then place in the *tsung fa* row (the number) 3087 above the *hsia fa* (row). For (the top row above the *shih*, called) *shang shang*, (first) put 10 above the given number of *pu* (for the segmental area), (i.e. the *shih*). Then place 450 (obtained by multiplying 45 in the *hsia fa* row by the first approximate root) below the given number of (segmental) area in *pu* (i.e. the *shih*) but above the *tsung fa* (row). This (row) is called the *fang fa*. Add together (the numbers) in the (two rows) of *fang (fa)* and *tsung fa* and multiplying (the sum) by the (first approximate root in the) *shang shang* (row), one obtains (the value) 35370. Dividing from (the *shih*) 46324, leaves a remainder 10954. Doubling the number in the *fang fa* row and adding it to the *tsung fa* gives 39870. This number is shifted back (to the right) by one digit. The (number in the) *hsia fa* (row) is also moved backward (by one step). Again place 2 (i.e. the second figure of the root) in the *shang shang* (row) (below the first figure 10). Multiplying the *hsia fa* (i.e. 45) (by 2) gives 90, which is placed in the *yü fa* (row) above the *hsia fa* (row). Adding this to 39870 gives 4077. Multiplying the (second figure in the) *shang shang* (row) by this number gives 8154, which when divided from 10954 (the number in the *shih* row) leaves a remainder 2800. Then double the *yü fa* and add this to the *tsung fang*, resulting in the number 4167. Multiply the remainder in the *shih* (row) by 45 to give 126,000, which is (now) placed in the *shih* (row). The *tsung fang* (row) (which contains the number 4167) is left unchanged, while the *hsia fa* is fixed as unity. Applying (the method of) *k'ai (p'ing) fang* (lit. 'extraction of square root') one obtains a numerator 30 (i.e. a fraction $\frac{30}{45}$), leaving a remainder 90 (on the counting-board). Dividing the numerator and the denominator by 15 (the fraction) becomes $\frac{2}{3}$ *pu*. Hence the answer ($12\frac{2}{3}$ *pu*)."

One of the problems on indeterminate equations quoted in YANG HUI's collection of ancient mathematical methods published in his book *Hsü Ku*

[36] 從而開方

Chai Ch'i Suan Fa [37] seems to have come from one of the missing problems of the *Chang Ch'iu-chien Suan Ching*. Referring to this problem YANG HUI said that both the *Chang Ch'iu-chien Suan Ching* and the *Pien Ku T'ung Yuan* [38] give only the 'Working' and that he had seen by chance manuscripts (of these texts). Thereupon he added to it the description of the 'Method' which he found missing. The Problem as shown in Figure 4 reads as follows¹³:—

"Each peck (*tou*) of high quality wine (*shun chiu*) costs 7 *kuan*, each peck of ordinary wine (*hsing chiu*) costs 3 *kuan* and 3 pecks of wine-dregs (*li chiu*) cost 1 *kuan*. Now if 10 *kuan* are used to buy 10 pecks of wine in all, find the amount (bought of each type of wine and the total money spent) for each."

"The 'Answer' says:

6 *shêng* of high quality wine costing 4 *kuan* 200 *wên*,

1 *tou* of ordinary wine costing 3 *kuan*

8 *tou* 4 *shêng* of wine-dreg costing 2 *kuan* 800 *wên*."

"The 'Method' says: Remove from the three different costs the cost of one of the items together with its amount. The remainder gives the combined cost of the other two items. (The individual costs) are found by using the method of *shuang iên shên shu* [39] (lit 'method of splitting a combined value into two') (a method identical to that used in solving simultaneous linear equations of two unknowns). Whenever fractions occur in the question multiply throughout by the common denominator."

"The 'Working' says: Place the values 10 *kuan* and 10 pecks of wine (on the counting-board). First subtract from them the amount 1 peck of ordinary wine and its value 3 *kuan*, giving a remainder of 7 *kuan*, being the combined cost of 9 pecks of high-quality wine and wine-dregs. Solving the problem following the method of *shuang iên shêng shu*, since 3 pecks of wine dregs cost 1 *kuan*, the values are multiplied throughout by the common denominator 3. The combined cost becomes 21 (*kuan*) and the cost 7 *kuan* for each peck of high quality wine is also multiplied by 3. 3 pecks of wine-dregs cost 1 *kuan*. Subtracting 9 pecks from the total cost leaves 12 *kuan*, which is to be placed at the *shih* position (on the counting-board) (i.e. dividend). Subtracting the respective costs for the high-grade wine and wine-dregs, leaves a remainder 20 *kuan*, which is to be placed in the *fa* position (on the counting-board) (i.e. divisor). Dividing (the *fa*) by the *shih* gives 6 pints (*shêng*) of high-grade wine. Subtracting this from 9 pecks gives 8 pecks 4 pints for the wine-dregs. Multiplying each by the cost gives the answer." See Figure 4.

The above amounts to solving the two simultaneous equations of 3 unknown.

$$x + y + z = 10 \dots\dots\dots (1)$$

$$7x + 3y + \frac{z}{3} = 10 \dots\dots\dots (2)$$

where *x*, *y* and *z* denote the amount of high-grade wine, ordinary wine and wine-dregs respectively. It gives one of the theoretically infinite number of

[37] 續古摘奇算法

[38] 辨玄通源

[39] 雙分身術

possible answers but fixes arbitrarily one of the values, in this case the amount of ordinary wine.

Equations (1) and (2) are thus reduced to the forms

$$x + z = 9 \dots\dots\dots (1)'$$

$$7x + \frac{z}{3} = 7 \dots\dots\dots (2)'$$

multiplying (2)' throughout by 3 gives

$$21x + z = 21$$

and subtracting equation (1)' from it gives

$$20x = 12$$

$$x = 0.6$$

Then $z = 9 - 0.6 = 8.4$.

Indeterminate analysis and indeterminate equations were always a marked mathematical interest to the Chinese mathematicians¹⁴. The last problem on the 'Hundred Fowls' in the *Chang Ch'iu-chien Suan Ching* deals with indeterminate analysis and has been widely discussed. In this particular case although the question in the problem is one concerning indeterminate analysis, in actual fact, the selection of an arbitrary value for one of the three unknowns has reduced the two equations to a pair of simultaneous linear equations of two unknowns, and thus the solution does not really deal with indeterminate analysis. Hence it is not unusual to find the 'Wines' problem placed at the beginning of the same chapter far apart from the 'Hundred Fowls' problem.

Finally, the problem in Chapter 3 of the *Chang Ch'iu-chien Suan Ching* immediately following the missing pages comes without its 'Question'. In the text that appears in the *Wan Yu Wên K'u* series the 'Question' has been reconstructed as shown in Figure 5. It reads:

"Five persons A, B, C, D and E are sharing 5 deer among themselves in the ratio of 6 : 5 : 4 : 3 : 2 respectively. Find the amount each receives."

Ch'ien Pao-tsung suggests that this was added by the Ch'ing mathematician Tai Chên^[40] (1724/1777)¹⁵.

The textual arrangements of the reconstructed Chinese text follow those of the 1777 edition of the *Chih Pu Tsu Chai Ts'ung Shu*. The text contains nine lines to a page and eighteen words to a line. The problem on the three grades of wine, together with the question on the distribution of deer do not completely fill up the empty pages 1a, 1b, 2a and 2b, leaving a space for probably one short but missing problem. This problem could have been originally placed either before or after that on the three grades of wine. Unfortunately, until now we have absolutely no clue regarding the nature of this missing problem¹⁶.

[40] 戴震

* The writer wishes to take this opportunity to thank Professor Dato Opperheim, Vice-Chancellor of the University of Malaya, and his colleagues Professor Wolfgang Franke, Mr. Cheng Hsi and Mr. Arthur W. E. Dolby for reading through the manuscript and for their valuable suggestions.

¹ This has been shown by Wang Ling from internal textual evidence. See WANG LING, *The 'Chiu Chang Suan Shu' and the History of Chinese Mathematics during the Han Dynasty*. Inaug. Diss. Cambridge 1956, and NEEDHAM, J., *Science and Civilisation in China*, vol. III, Cambridge 1959, p. 33. The *Chang Ch'iu-chien Suan Ching* is also discussed in CH'EN PAO-TSUNG, *Chung-kuo Suan Hsüeh Shih*^[41] vol. I, Peking 1932, p. 51 to p. 52. CH'EN PAO-TSUNG, *Suan Ching Shih Shu*^[42], vol. 2, Peking 1963, p. 325, gives the year limits as A.D. 466 to A.D. 485.

² See *Ch'in Ting T'ien Lu Ling Lang Shu Mu Hou Pien*^[43] ch. 1, p. 7a to p. 14a.

³ LI YEN^[44], *Chung-kuo Shu Hsüeh Ta Kang*^[45], vol. I, Shanghai 1931, p. 98 gives the year 1781.

⁴ CH'EN PAO-TSUNG, *Suan Ching Shih Shu*, vol. II, Peking 1963, p. 325 ff.

⁵ See LI YEN, *Chung-kuo Shu Hsüeh Ta Kang*, vol. I, Shanghai 1931, p. 47. and LI YEN, *Chung-kuo Suan Hsüeh Shih*, Shanghai 1937, reprinted, 1955, p. 25.

⁶ Improved formulae were given by Shên Kua^[46] in the eleventh-century and Kuo Shou-ching^[47] in the thirteenth century. See for example NEEDHAM, J., *Science and Civilisation in China*, Vol. III, Cambridge 1959, p. 38 and p. 39. See also LI YEN, *Chung-kuo Suan Hsüeh Shih*, Shanghai 1937, reprinted 1955, p. 134, for other Chinese formulae.

⁷ See LI YEN *Chung-kuo Suan Hsüeh Shih*, Shanghai 1937, reprinted 1955, p. 25.

⁸ In Chapter 4, pp. 39b and 40a in the Kiangsu Book Company (*Chiung-su Shu chü*) edition.

⁹ See LI YEN, *Chung-kuo Suan Hsüeh Ta Kang*, vol. I, Shanghai 1931, p. 43.

¹⁰ See f. n. 5 above.

¹¹ See NEEDHAM, J., *Science and Civilisation in China* vol. III, Cambridge 1959, p. 126, and also WANG LING & NEEDHAM, *Horner's Method in Chinese Mathematics: its origine in the Root-Extraction Procedures of the Han Dynasty*, *T'oung Pao*, 1955, vol. 43, p. 345.

¹² Division as employed in this yields the same result as subtraction. In fact the term *ch'ü*^[48] may be taken to mean 'take away' or 'remove'.

¹³ From a hand written copy made by Seki Kōwa in the year 1661, from the Korean edition (1433) of the *Hsü Ku Chai Ch'i Suan Fa*, ch. 2 pp. 3a & 3b. and through the courtesy of the late Mr. LI YEN.

¹⁴ For further discussions of this topic see for example LI YEN, *Chung Suan Shih Lun Ts'ung*^[49], *Gesammelte Abhandlungen über die Geschichte der chinesischen Mathematik*, vol. 1, 1947, Shanghai p. 61, CH'EN PAO-TSUNG, *Ku Suan K'ao Yüan*^[50] *Über den Ursprung der chinesischen Mathematik*, Shanghai 1930, pp. 45 ff, HSÜ SHUN-FANG^[51], *Ku Suan Fa Chih Hsin Yen Chiu*^[52], Shanghai 1935 pp. 1—28, and NEEDHAM, J. *Science and Civilisation in China*, vol. III, Cambridge 1959, pp. 119 ff.

¹⁵ See CH'EN PAO-TSUNG, *Suan Ching Shih Shu*, vol. 2, Peking 1963, p. 373.

¹⁶ For example, there is no mention of the *Chang Ch'iu-chien Suan Ching* in the fragment of the *Yung Lo Ta Tien* chapters 16343 and 16344, now available to us.

[41] 中國算學史

[42] 算經十書

[43] 欽定天祿琳琅目後編

[44] 李儼

[45] 中國數學大綱

[46] 沈括

[47] 郭守敬

[48] 除

[49] 中算史論叢

[50] 古算考源

[51] 許繩功

[52] 古算法之新研究

今有弧田弦六十八步五分步之三爲田二畝
 三十四步四十五分步之三十二間矢幾何
 答曰矢一十二步三分步之二
 術曰置田積步倍之爲實以弦步數爲從

而開方除之卽得矢
 草曰置二畝以步法二百四十乘之內子
 三十四得五百一十四步以分母四十五
 乘之內子三十二得二萬三千一百六十
 二倍之得四萬六千三百二十四又置弦
 步數六十八步以分母五乘之內子三得
 三百四十三以弦步數分母五除田積步
 分母四十五得九以九乘三百四十三得
 三千八十七爲從以從法開方除置前積

Figure 3 (b)
 Reconstruction of p. 23a

Figure 3 (a)
 Last four lines of p. 22b

四萬六千三百二十四於上別置分母四
 十五於下常超一位步至百止又置從法
 三千八十七於下法之上以上商置一十
 於積步之上又置四百五十於積步之下
 從法之上名曰方法併方從法以命上商
 得三萬五千三百七十除四萬六千三百
 二十四餘一萬九百五十四又倍方法而
 併從方得三萬九千八百七十退一位下
 法再退又置二於上商之下又置九十於

Figure 3 (b)
 Reconstruction of p. 23 b

下法之上名曰隅法併三千九百八十七
 得四千七十七以命上商得八千一百五
 十四除一萬九百五十四餘二千八百又
 倍隅法而併從方得四千一百六十七以
 下法四十五爲母乘餘實二千四百得十
 二萬六千爲實從方不動下法定爲一而
 開方除之得三十爲子不盡九子母各以
 十五約之爲三分步之二十合問

Figure 3 (c)
 Reconstruction of p. 24 a

漢中郡守前司隸臣甄鸞注經

唐朝議大夫行太史令上輕車都尉臣李淳風等

奉敕注釋

唐算學博士臣劉孝孫細草

醇酒每斛七貫行酒每斛三貫醪酒三斛直一貫今支
一十貫買酒十斛問各得幾何

答曰醇酒六升價四貫二百文

行酒一斛價三貫文

醪酒八斛四升價二貫八百文

術曰三價中以一價除出一位所得之數其餘二
物共價如雙分身法求之題有分子者通之
草曰置十貫酒十斛先以行酒一斛三貫除出一
斛餘錢七貫即醇酒九斛共價也如雙分身術求
之內醪酒三斛直一貫合通分以共價七貫三因
一醇酒一斛直七貫亦用醪酒三斛直一貫以醇
酒一貫乘九斛減共錢餘一十二貫為實以醇醪
二價相減餘二十貫為法除實行醇酒六升反減

九斛共數得醪酒八斛四升以各價乘之合問

Figure 4 Possibly pages 1a, 1b and part of 2a

今有甲乙丙丁戊五人共分五鹿欲以六五四三二差
之間各得幾何

Figure 5
Part of page 2b, Chapter 3