The lost problems of the Chang Ch'iu-chien Suan Ching, a fifth-century Chinese mathematical manual

by Ho Peng Yoke[1]

(University of Malaya, Kuala Lumpur, Malaysia)

Among the few traditional Chinese mathematical texts that still survive today is the Chang Ch'iu-chien Suan Ching[2] (Mathematical Manual of Chang Ch'iu-chien), written by Chang Ch'iuchien[3] some time between the years A.D. 468 and A.D. 486[4]. At the beginning of the Ch'ing period Wang Chieh[5] had in his possession a Southern Sung (1127 to 1279) printed copy of this text. Later it was acquired by Mao Chin[6], and now it is preserved in the Shanghai Library. During the first year of the K'ang-hsi reign-period (1662) Mao I[7] made a copy from the Sung edition, and this later entered the T'ien Lu Lin Lang Ko[8] Palace-Library[9]. On this were based the edition of the Chang Ch'iu-chien Suan Ching in the Ssu K'u Ch'üan Shu Collection and that in K'ung Chi-han's[10] (1739/1783) Wei Po Hsieh Ts'ung Shu[11] Collection (1773). The texts of the Chang Ch'iu-chien Suan Ching later included in the Chih Pu Tsu Chai Ts'ung Shu[12] Collection (1777)[13], the Ku Chin Suan Hsüeh Ts'ung Shu[14] Collection (1898) and the Wan Yu Wen K'u (1937) all followed the Wei Po Hsieh Ts'ung Shu. The first series of the Ts'ung Shu Chi Ch'eng (1939) reproduces the text from the Chih Pu Tsu Chai Ts'ung Shu. Textual comparisons have been recently made by Ch'ien Pao-tsung[15].

The text of the Chang Ch'iu-chien Suan Ching that we now have in all the versions mentioned above is not complete. The missing portion comes at the end of the Second Chapter and the beginning of the Third Chapter in all these cases. The end of the Second Chapter in the Chih Pu Tsu Chai Ts'ung Shu edition, for example, consists of one or more blank leaves, beginning from page 22a, while the beginning of the Third Chapter consists of two blank leaves, representing pages 1a, 1b, 2a and 2b. Page 1a of the Third Chapter contains five printed lines giving the title of the book, the name of the author, the name of the commentator CH'EN LUAN[16] (fl. A.D. 560/A.D. 580) and the name of the writer of the workings, LIU HSIAO-SUN[17] (A.D. 576/A.D. 625). It is the purpose of this article to discuss the missing part of the text.
The fifty-fourth problem in the *Chang Ch’iu-chien Suan Ching* deals with the finding of the sagitta of a segmental area (hu t’ien)\(^{15}\), given the area and the length of the chord. The whole Working of this example is missing, and it must have been contained in page 22a and page 22b of Chapter Two. Similarly, the last few words of the given ‘Method’ must have been formerly contained in page 22a. This is rather unfortunate, as the ‘Working’ is supposed to explain the *tai tsung k’ai lang*\(^{16}\) method employed by Chinese mathematicians for solving quadratic equations and equations of higher degree\(^5\). What still remains of this Problem says:

> "Given a segment with a chord 68 \(\frac{3}{5}\) pu\(^{17}\) long and area 2 mou\(^{18}\) \(\frac{32}{45}\) pu, find its sagitta."

The ‘Answer’ says: The sagitta is 12\(\frac{2}{3}\) pu.

The ‘Method’ says: Place twice the segmental area expressed in (square) pu (on the calculation board) in the *shih*\(^{19}\) (position).

Take the number of pu in the chord as the *tsung*\(^{20}\) ...

The ‘Method’ is incomplete and the ‘Working’ is missing altogether. Nonetheless, one can see that Chang Ch’iu-chien must have in mind the same formula used by Liu Hui\(^{21}\) some two centuries earlier for a relationship between the area \(A\), the chord \(c\) and the sagitta \(s\) of a segment, i.e.

\[
A = \frac{1}{2} s \times (s + c)
\]

This formula is of course not quite correct\(^6\). However, it is the method of solving the quadratic equation involved that is of particular interest to us. It must be also on the basis of this same formula that Li Yen amended the last fraction of the segmental area from \(\frac{22}{45}\) to \(\frac{31}{45}\), assuming an exact solution\(^7\).

The ‘Working’ has been reconstructed by Ku Kuân-küâng\(^{22}\) (1799/1862) in his *Chiu Shu T’s’un Ku*\(^{23}\)\(^8\). The passage is as shown in Figure 1. It reads as follows:

> "The Method says: Place twice the segmental area expressed in (square) pu (on the counting board) in the *shih* (position). Take the number of pu of the chord as the *tsung*. The sagitta is obtained by means of (the method of) *k’ai p’êng lang*\(^{24}\) (lit. 'extraction of square root'). (Note:) the last sentence above has been reconstructed from the context."

The ‘Working’ says: Use the equivalent of mou and pu measures and convert 2 mou to 480 pu. Add to this the other pu values (i.e. \(\frac{32}{45}\)). Then change into improper fraction by multiplying (514) by the denominator (45) and add to it the numerator (32), (thus) obtaining \(\frac{23162}{45}\) with a numerator 23162. Doubling this value gives 46324, which is (to be placed in) the *shih* (position on the counting-board). Also multiply the chord of 68 pu by the denominator 45, giving 3600 (and place this number somewhere) in a top position (on the counting-board), and multiply the numerator 3 by 9, giving 27. (Note that) 45 is equal to 9 times 5, hence (the numerator 3 in the fraction \(\frac{3}{5}\) is) multiplied by 9 (instead of multiplying the fraction itself by 45).

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\(^{15}\) 弧弦

\(^{16}\) 織從開方

\(^{17}\) 步

\(^{18}\) 歎

\(^{19}\) 實

\(^{20}\) 從

\(^{21}\) 劉徽

\(^{22}\) 顧觀光

\(^{23}\) 九數存古

\(^{24}\) 開平方

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Figure 1 Reconstruction by Ku Kuan-kuang
(The value 27) is added to (the figure 3600 in) the top position, giving the number 3087 which is (then placed in) the tsung fang [25] (position on the counting-board). (The number) 45 is (placed in) the chêng yü [26] (position on the counting-board). Applying (the method of) k'ai p'ing fang (lit. 'extraction of square root') (the result) 12 pu is obtained, with remainders 2800 in the shih (position), 4167 in the (tsung) lang (position) and 45 in the (chêng) yü (position). Taking (the value) 45 in the (chêng) yü (position) as the denominator, multiply the remainder in the shih (position) by this number to give 126,000 which is (now) taken as the shih. The (number in the tsung) lang (position) is left unchanged, while the (number in the chêng) yü (position) is fixed as unity. Applying (the method of) k'ai p'ing fang (lit. extraction of square root) one obtains a numerator 30 (i.e. a fraction \( \frac{30}{45} \)). (Note that) the remainder 990 is left (on the counting-board). Dividing the numerator and the denominator by 15 (the fraction) becomes \( \frac{2}{3} \) pu. Hence the answer (12 \( \frac{2}{3} \) pu).

It can be seen that Ku Kuan-kuang first expressed the problem in the form of a quadratic equation:

\[
45s^2 + 3087s - 46324 = 0
\]

He gives the result without explaining the process. From his result we can deduce that he has used a process well-known to the thirteenth-century Chinese mathematicians Ch'in Chiu-shao [27], Li Chih [28] and Yang Hui [29] and later developed independently in the early nineteenth century by Ruffini and Horner. The process would be as follows:

<table>
<thead>
<tr>
<th>chêng yü</th>
<th>tsung fang</th>
<th>shih</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>3087</td>
<td>-46324</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>450</td>
<td>3537</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-10954</td>
<td></td>
</tr>
<tr>
<td></td>
<td>450</td>
<td>3987</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>chêng yü</th>
<th>tsung fang</th>
<th>shih</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>3987</td>
<td>-10954</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>8154</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2800</td>
</tr>
<tr>
<td></td>
<td>4077</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4167</td>
<td></td>
</tr>
</tbody>
</table>

A misprint appears in Ku Kuan-kuang's notes on finding the fraction. After converting the equation

\[
45s^2 + 4167s - 2800 = 0
\]

into the form

\[
y^2 + 4167y - 126,000 = 0
\]
where \( s = \frac{y}{45} \).

The process of finding \( y \) continues as follows:

<table>
<thead>
<tr>
<th>chêng yû</th>
<th>tsung lang</th>
<th>shih</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4167</td>
<td>126000</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>125910</td>
</tr>
<tr>
<td></td>
<td>4197</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4227</td>
<td></td>
</tr>
</tbody>
</table>

The value \( y = 30 \) or \( s = \frac{30}{45} = \frac{2}{3} \) is obtained.

The remainder should read '90' and not '990' as stated in Ku Kuan-Kuang's notes.

The process used above was already in use during the thirteenth century in China and was described both by Ch'in Chiu-shao and Yang Hui. The former, for example, called it the lien chih t'ung t'i shu\(^{[10]}\). It is of course possible to obtain the same result by applying the process of lien chih t'ung t'i shu from the very beginning by converting the equation

\[
45s^2 + 3087s - 46324 = 0
\]

into the form

\[
y^2 + 3087y - 2084580 = 0
\]

where \( s = \frac{y}{45} \).

Li Yen is not entirely satisfied with Ku Kuan-Kuang's reconstruction of the 'Workings' in the problem on the segmental area in the Chang Ch'iu-chien Suan Ch'ing\(^{[9]}\). Earlier he mentions the sad omission of the tai tsung k'ai lang method at this point in the text, and this method as we have seen, is only referred to by Ku Kuan-Kuang as the k'ai p'ing lang method, without any explanation of the process\(^{[19]}\). Wang Ling and Needham say that it is possible to show that if the text of the Chiu Chang Suan Shu\(^{[81]}\) is very carefully followed, the essentials of the methods used by the Chinese for solving numerical equations of the second and higher orders and similar to that developed by Horner in 1819, are already there at a time which may be dated as of the first century B.C.\(^{[11]}\). It may therefore be assumed that the method was known to Chang Ch'iu-chien in the A.D. fifth century and to Liu Hsiao-sun in the sixth and seventh centuries. In fact, this can be seen from Problem No. 51 in the Chang Ch'iu-chien Suan Ch'ing, which gives the solution of a second degree equation in the form \( x^2 - 127449 = 0 \). It is worthwhile to give a detailed analysis of the process given.

The problem involves the finding of the side of a square farm with a given area 127449 (square) pu. The steps in the 'Working' are as follows:

**Step 1**

置前積步數於上

Place the given (number expressed in) number of (square) pu in an upper (row of the counting-board). (This is called the shih).

\(^{[10]}\) 遼枝同體術

\(^{[21]}\) 九章算術
Step 2
Make use of one counting-rod in the bottom (row of the counting-board in the furthest right-hand digit column).

Step 3
This one counting-rod is always moved forward (from right to left) by steps of two places until it reaches the hundredth place-value (in this case). (This row is called the \textit{hsia \textit{fa}})\textsuperscript{[32]}.

Step 4
For (the top row above the \textit{shih}, called) \textit{shang shang}\textsuperscript{[33]}, (first) put 300 above the given number (i.e. the \textit{shih}).

Step 5
Then place 30,000 (obtained by multiplying the \textit{hsia \textit{fa}} by the first approximate root) below the given number (i.e. the \textit{shih}) but above the \textit{hsia \textit{fa}}. This (row) is called the \textit{fang \textit{fa}}\textsuperscript{[34]}. Multiplying by (the digit of the first root in) the \textit{shang shang} gives $3 \times 3(0,000) = 9(0,000)$.

Step 6
90,000 is now divided from (the \textit{shih}, and the same row is now replaced by the remainder)\textsuperscript{[12]}.

Step 7
Then double the \textit{fang-\textit{fa}} and move it back by one digit (to the right).

Step 8
The (counting-rod in the) \textit{hsia \textit{fa}} (row) is also moved backward (by one step).

Step 9
Again place 50 (i.e. the second figure of the root) below the \textit{shang shang} row.

Step 10
Then again place 500 (obtained by multiplying the \textit{hsia \textit{fa}} by the second figure of the root) above the \textit{hsia \textit{fa}}. This is called the \textit{yù \textit{fa}}\textsuperscript{[35]}.

\textsuperscript{[32]} 下法 \hspace{1cm} [32] 上商 \hspace{1cm} [34] 方法 \hspace{1cm} [35] 隅法
Step 11
以方隅二法除實 餘有四千九百四十九
The shih is divided from (the product of the second figure of the root, i.e. 5 and the sum of) the tang (fa) and the yü (la), yielding a remainder 4949.

Step 12
又倍隅法以併方 得七千 退一等
Then double the yü la and add this to the tang (fa), giving 7,000. This is moved backward by one place.

Step 13
下法再退
The (counting-rod in the) hsia la row is again moved backward (by one step).

Step 14
又置七於上高五十之下
Again place 7 (i.e. the third figure of the root) below (the second figure of the root) 50 in the shang shang row.

Step 15
又置七於下法之上 名曰隅法
Then again place 7 (obtained by multiplying the hsia la by the third figure of the root) above the hsia la. This is called the yü la.

Step 16
以方隅法二除實 得合前問
Dividing the shih by the (product of the third root figure and the) sum of the tang (fa) and yü (la), the answer corresponding to the above question is obtained.

As a comparison the corresponding steps in the traditional Chinese method as described by Liu Hsiao-sun and the Ruffini-Horner method are shown in Figure 2 (a) to (e).

Figure 2
(a)
(b) shang shang (step 4)

shih

fang ia (1st part of step 5)

hsia ia (step 3)

(c) shang shang

shih

(2nd part of step 5)

fang ia

hsia ia

(d) shang shang (step 9)

shih (step 6)

(step 7)

yü ia (step 10)

hsia ia (step 8)
A striking resemblance between the two methods can be noticed.

An attempt is here made to improve on the re-construction of the lost text of Ku Kuan-Kuang by incorporating the tai-tsung k'ai fang method and by copying the literary style of Liu Hsiao-sun, the author of the 'Working' in the Chang Ch'iuk-chien Suan Ching. The reconstructed Chinese text as shown in Figure 3 reads:

"Given a segment with a chord $68\frac{3}{5}$pu long and area $2$ mou $34\frac{32}{45}$ pu find its sagitta.

The 'Answer' says: The sagitta is $12\frac{2}{3}$pu.

The 'Method' says: Place twice the segmental area expressed in (square) pu (on the calculation board) in the shih (position). Take the number of pu of
the chord as the tsung. The sagitta is obtained by means of (the method) tsung êrh k’ai tang \textsuperscript{[36]} (i.e. t'ai tsung k’ai tang).

The 'Working' says: Place (the number) 2 mou (on the calculation board) and multiply it by (the conversion factor) 240 (to bring it to) pu. Adding the number 34 gives 514 pu. Multiplying by the denominator 45 and adding the numerator 32 gives 23,162. Twice this number gives 46,324. Then place the value of the chord expressed in number of pu, (i.e.) 68 pu (on the counting-board), multiply it by the denominator 5 and add the numerator 3, giving 343. Dividing the denominator for the segmental area (expressed in square) pu, (i.e.) 45 by the denominator for the chord length (expressed in pu, (i.e.) 5 gives 9. Multiplying 343 by 9 gives 3087, which is to be taken as the tsung. Then employ the method of tsung la k’ai tang to extract the root."

"Place the above number 46,324 for the area in an upper (row of the counting-board). (This is called the shih). Place the denominator 45 separately in the bottom (row of the counting-board in the furthest right-hand digit column). This number is always moved forward (from right to left) by steps of two places until it reaches the hundredth place (in this case). (This row is called the hsia la). Then place in the tsung la row (the number) 3087 above the hsia la (row). For (the top row above the shih, called) shang shang, (first) put 10 above the given number of pu (for the segmental area), (i.e. the shih). Then place 450 (obtained by multiplying 45 in the hsia la row by the first approximate root) below the given number of (segmental) area in pu (i.e. the shih) but above the tsung la (row). This (row) is called the lang la. Add together (the numbers) in the (two rows) of lang (la) and tsung la and multiplying (the sum) by the (first approximate root in the) shang shang (row), one obtains (the value) 35370. Dividing from (the shih) 46324, leaves a remainder 10954. Doubling the number in the lang la row and adding it to the tsung la gives 39870. This number is shifted back (to the right) by one digit. The (number in the) hsia la (row) is also moved backward (by one step). Again place 2 (i.e. the second figure of the root) in the shang shang (row) (below the first figure 10). Multiplying the hsia la (i.e. 45) (by 2) gives 90, which is placed in the yü la (row) above the hsia la (row). Adding this to 39870 gives 4077. Multiplying the (second figure in the) shang shang (row) by this number gives 8154, which when divided from 10954 (the number in the shih row) leaves a remainder 2800. Then double the yü la and add this to the tsung lang, resulting in the number 4167. Multiply the remainder in the shih (row) by 45 to give 126,000, which is (now) placed in the shih (row). The tsung lang (row) (which contains the number 4167) is left unchanged, while the hsia la is fixed as unity. Applying (the method of) K’ai (p’ing) tang (lit. 'extraction of square root') one obtains a numerator 30 (i.e. a fraction \(\frac{30}{45}\)), leaving a remainder 90 (on the counting-board). Dividing the numerator and the denominator by 15 (the fraction) becomes \(\frac{2}{3}\) pu. Hence the answer (12\(\frac{2}{3}\) pu)."

One of the problems on indeterminate equations quoted in Yang Hui's collection of ancient mathematical methods published in his book Hsü Ku
Chai Ch'i Suan Fa\textsuperscript{[37]} seems to have come from one of the missing problems of the Chang Ch'iui-chien Suan Ching. Referring to this problem Yang Hui said that both the Chang Ch'iui-chien Suan Ching and the Pien Ku T'ung Yuan\textsuperscript{[38]} give only the 'Working' and that he had seen by chance manuscripts (of these texts). Thereupon he added to it the description of the 'Method' which he found missing. The Problem as shown in Figure 4 reads as follows\textsuperscript{13}:

"Each peck (tou) of high quality wine (shun chiu) costs 7 kuan, each peck of ordinary wine (hsing chiu) costs 3 kuan and 3 pecks of wine-dregs (li chiu) cost 1 kuan. Now if 10 kuan are used to buy 10 pecks of wine in all, find the amount (bought of each type of wine and the total money spent) for each."

"The 'Answer' says:

6 sheng of high quality wine costing 4 kuan 200 wen,
1 tou of ordinary wine costing 3 kuan
8 tou 4 sheng of wine-dreg costing 2 kuan 800 wen."

"The 'Method' says: Remove from the three different costs the cost of one of the items together with its amount. The remainder gives the combined cost of the other two items. (The individual costs) are found by using the method of shuang l'en shen shu\textsuperscript{[39]} (lit 'method of splitting a combined value into two') (a method identical to that used in solving simultaneous linear equations of two unknowns). Whenever fractions occur in the question multiply throughout by the common denominator."

"The 'Working' says: Place the values 10 kuan and 10 pecks of wine (on the counting-board). First subtract from them the amount 1 peck of ordinary wine and its value 3 kuan, giving a remainder of 7 kuan, being the combined cost of 9 pecks of high-quality wine and wine-dregs. Solving the problem following the method of shuang l'en shen shu, since 3 pecks of wine dregs cost 1 kuan, the values are multiplied throughout by the common denominator 3. The combined cost becomes 21 (kuan) and the cost 7 kuan for each peck of high quality wine is also multiplied by 3. 3 pecks of wine-dregs cost 1 kuan. Subtracting 9 pecks from the total cost leaves 12 kuan, which is to be placed at the shih position (on the counting-board) (i.e. dividend). Subtracting the respective costs for the high-grade wine and wine-dregs, leaves a remainder 20 kuan, which is to be placed in the ta position (on the counting-board) (i.e. divisor). Dividing (the ta) by the shih gives 6 pints (sheng) of high-grade wine. Subtracting this from 9 pecks gives 8 pecks 4 pints for the wine-dregs. Multiplying each by the cost gives the answer." See Figure 4.

The above amounts to solving the two simultaneous equations of 3 unknown.

\[x + y + z = 10\]  \hspace{1cm} (1)
\[7x + 3y + \frac{2}{3}z = 10\]  \hspace{1cm} (2)

where x, y and z denote the amount of high-grade wine, ordinary wine and wine-dregs respectively. It gives one of the theoretically infinite number of

\textsuperscript{[37]} 續古摘奇算法  \hspace{1cm} \textsuperscript{[38]} 辨古通源  \hspace{1cm} \textsuperscript{[39]} 雙分身術
possible answers but fixes arbitrarily one of the values, in this case the amount of ordinary wine.

Equations (1) and (2) are thus reduced to the forms

\[ x + z = 9 \]  \hspace{1cm} (1)'
\[ 7x + \frac{3}{3} = 7 \]  \hspace{1cm} (2)'

multiplying (2)' throughout by 3 gives

\[ 21x + z = 21 \]

and subtracting equation (1)' from it gives

\[ 20x = 12 \]
\[ x = 0.6 \]

Then \[ z = 9 - 0.6 = 8.4. \]

Indeterminate analysis and indeterminate equations were always a marked mathematical interest to the Chinese mathematicians\(^{14}\). The last problem on the 'Hundred Fowls' in the *Chang Ch'iu-chien Suan Ching* deals with indeterminate analysis and has been widely discussed. In this particular case although the question in the problem is one concerning indeterminate analysis, in actual fact, the selection of an arbitrary value for one of the three unknowns has reduced the two equations to a pair of simultaneous linear equations of two unknowns, and thus the solution does not really deal with indeterminate analysis. Hence it is not unusual to find the 'Wines' problem placed at the beginning of the same chapter far apart from the 'Hundred Fowls' problem.

Finally, the problem in Chapter 3 of the *Chang Ch'iu-chien Suan Ching* immediately following the missing pages comes without its 'Question'. In the text that appears in the *Wan Yu Wen K'u* series the 'Question' has been reconstructed as shown in Figure 5. It reads:

"Five persons A, B, C, D and E are sharing 5 deer among themselves in the ratio of 6 : 5 : 4 : 3 : 2 respectively. Find the amount each receives."

Ch'ien Pao-tsung suggests that this was added by the Ch'ing mathematician Tai Chen\(^{40}\) (1724/1777)\(^{15}\).

The textual arrangements of the reconstructed Chinese text follow those of the 1777 edition of the *Chih Pu Tsu Chai Ts'ung Shu*. The text contains nine lines to a page and eighteen words to a line. The problem on the three grades of wine, together with the question on the distribution of deer do not completely fill up the empty pages 1a, 1b, 2a and 2b, leaving a space for probably one short but missing problem. This problem could have been originally placed either before or after that on the three grades of wine. Unfortunately, until now we have absolutely no clue regarding the nature of this missing problem\(^{18}\).
Foot-Notes

1 The writer wishes to take this opportunity to thank Professor Dato Oppenheim, Vice-Chancellor of the University of Malaya, and his colleagues Professor Wolfgang Franke, Mr. Cheng Hsi and Mr. Arthur W. E. Dolby for reading through the manuscript and for their valuable suggestions.

1 This has been shown by Wang Ling from internal textual evidence. See WANG LING, The 'Chiu Chang Suan Shu' and the History of Chinese Mathematics during the Han Dynasty. Inaug. Diss. Cambridge 1956, and NEEDHAM, J., Science and Civilisation in China, vol. III, Cambridge 1959, p. 33. The Chang Ch’iu-chien Suan Ching is also discussed in CH’IEN PAO-TSUNG, Chung-kuo Suan Hsüeh Shih [41] vol. I, Peking 1932, p. 51 to p. 52. CH’IEN PAO-TSUNG, Suan Ching Shih Shu [42], vol. 2, Peking 1963, p. 325, gives the year limits as A.D. 466 to A.D. 485.


3 This has been shown by Wang Ling from internal textual evidence. See WANG LING, The 'Chiu Chang Suan Shu' and the History of Chinese Mathematics during the Han Dynasty. Inaug. Diss. Cambridge 1956, and NEEDHAM, J., Science and Civilisation in China, vol. III, Cambridge 1959, p. 33. The Chang Ch’iu-chien Suan Ching is also discussed in CH’IEN PAO-TSUNG, Chung-kuo Suan Hsüeh Shih [41] vol. I, Peking 1932, p. 51 to p. 52. CH’IEN PAO-TSUNG, Suan Ching Shih Shu [42], vol. 2, Peking 1963, p. 325, gives the year limits as A.D. 466 to A.D. 485.


6 In Chapter 4, pp. 39 b and 40 a in the Kiangsu Book Company (Chiang-su Shu chu) edition.

7 See Li Yen Chung-kuo Suan Hsüeh Shih, Shanghai 1937, reprinted 1955, p. 25.

8 Division as employed in this yields the same result as subtraction. In fact the term ch’u [48] may be taken to mean 'take away' or 'remove'.

9 From a hand written copy made by Seki Kōwa in the year 1661, from the Korean edition (1433) of the Hsü Ku Chai Ch’i Suan Fa, ch. 2 pp. 3 a & 3 b. and through the courtesy of the late Mr. Li Yen.


11 Division as employed in this yields the same result as subtraction. In fact the term ch’u [48] may be taken to mean 'take away' or 'remove'.

12 From a hand written copy made by Seki Kōwa in the year 1661, from the Korean edition (1433) of the Hsü Ku Chai Ch’i Suan Fa, ch. 2 pp. 3 a & 3 b. and through the courtesy of the late Mr. Li Yen.


14 For example, there is no mention of the Chang Ch’iu-chien Suan Ching in the fragment of the Yung Lo Ta Tien chapters 16343 and 16344, now available to us.
Figure 3 (b)
Reconstruction of p. 23a

Figure 3 (a)
Last four lines of p. 22b
Figure 3 (c)
Reconstruction of p. 24a

Figure 3 (b)
Reconstruction of p. 23b
Figure 4 Possibly pages 1a, 1b and part of 2a
今有甲乙丙丁戊五人共分五鹿欲以六五四三四三二差
之間各得幾何

Figure 5
Part of page 2b, Chapter 3